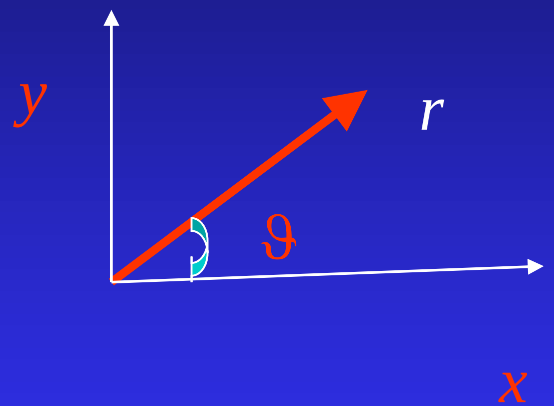
Chapter 3 Vectors

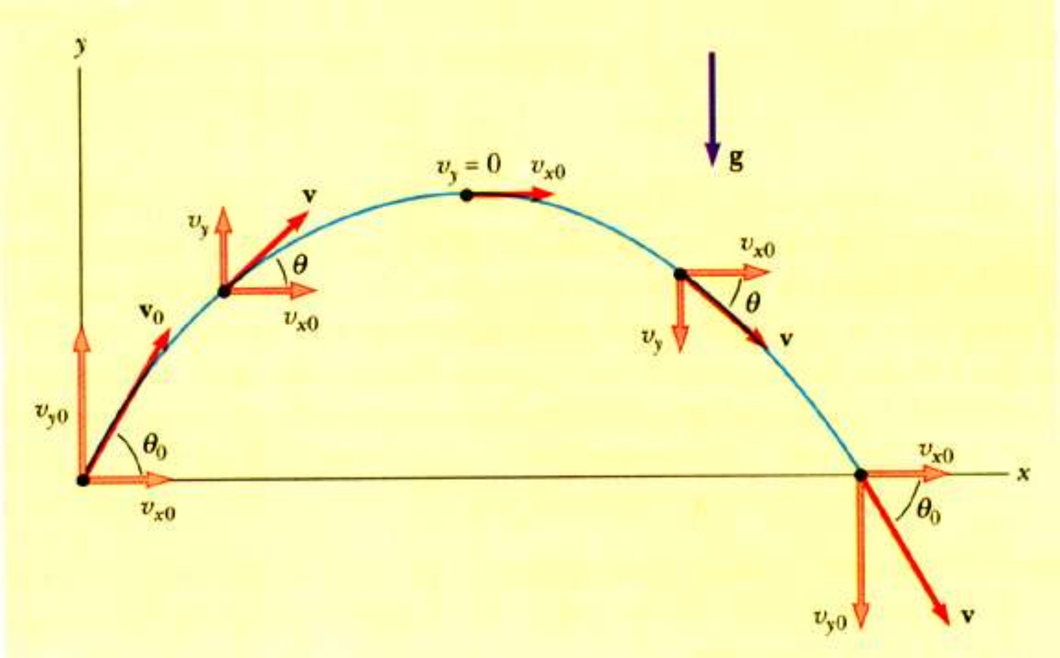
- Physics deals with many quantities that have both
  - ***Size***
  - ***Direction***



(x,y) (r,ϑ)

- **VECTORS !!!!!**
- E.g. Displacement, Velocity, Acceleration, Force, Torque

x and *y components* of motion are independent-"LINEARITY"



Componets

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$\vec{v} = (v_x, v_y) = v_x\,\hat{i} + v_y\,\hat{j}$$

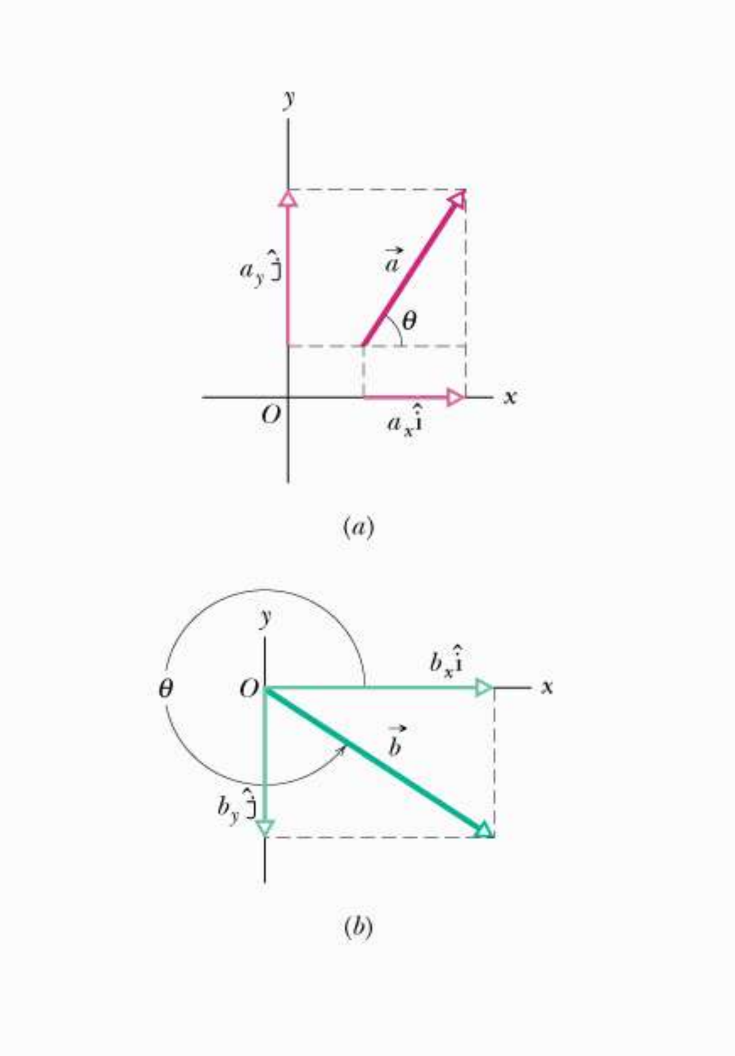
Unit vectors

Definitions

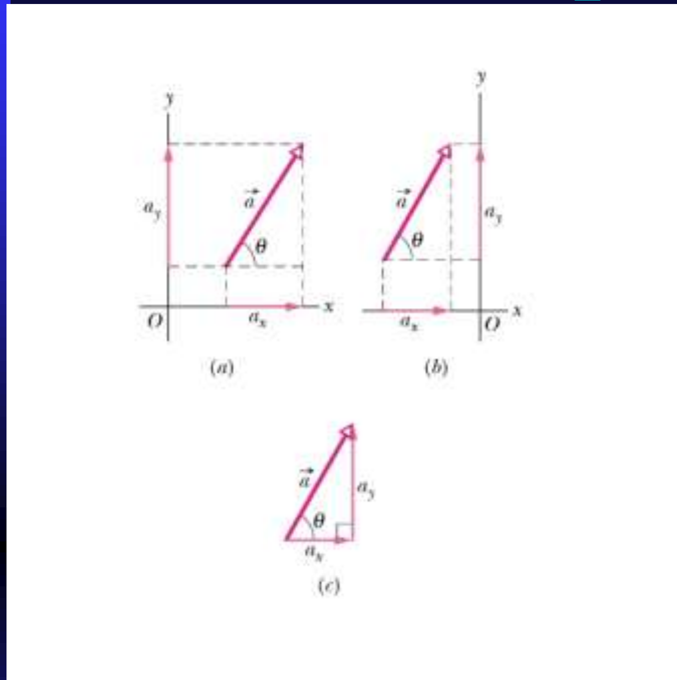
- **SCALAR** provides information about how large a measurement is
  - Gives one item of information, magnitude, temperature=T
- **VECTOR** provides information about how large a measurement is and the direction of the measurement
  - ***Ordered pair numbers required*** (x,y) (r,θ)

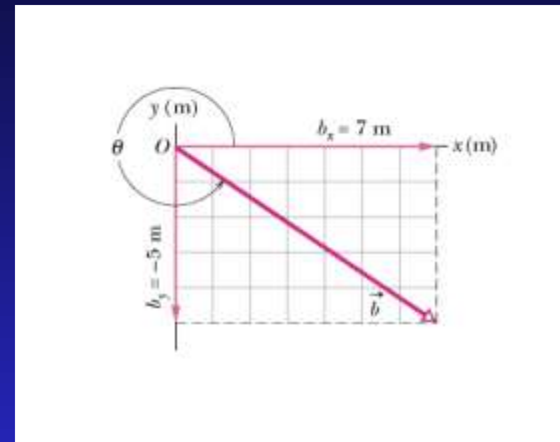
Vectors are added or subtracted according

- *Ordered pair numbers* (x,y) (r,ϑ)



Vector Components





$$\cos\theta = \frac{a_x}{\sqrt{a_x^2 + a_y^2}}$$

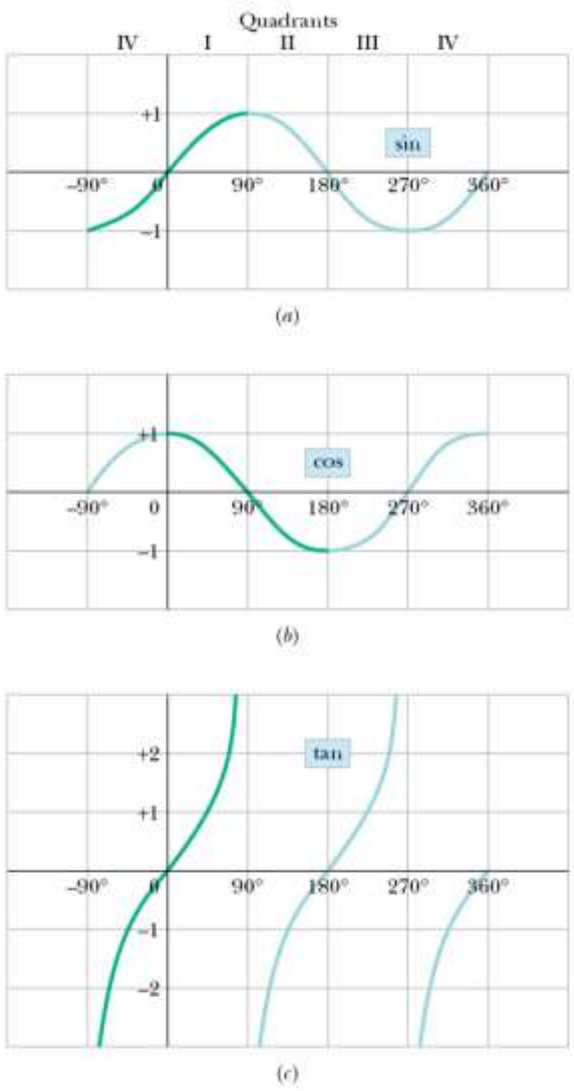
$$\sin\theta = \frac{a_y}{\sqrt{a_x^2 + a_y^2}}$$

$$\tan\theta = \frac{a_y}{a_x}$$

If you use your calculator

to determine angle you will find

$\tan^{-1}(-5/7) = -35^{o} \rightarrow 325^{o}$


Quadrants
IV
I
II
III
IV
+1
sin
-90°
0
90°
180°
270°
360°
−1
(a)
+1
cos
-90°
0
90°
180°
270°
360°
−1
(b)
+2
tan
+1
-90°
0
90°
180°
270°
360°
−1
−2
(c)

VECTOR ALGEBRA

- **Scalar multiplication**

$$\vec{b} = 2 \cdot \vec{a}$$

- Vectors are **added or subtracted** according to the rules for ordered pairs

- **(a_x, a_y)-(b_x, b_y) coordinates!**

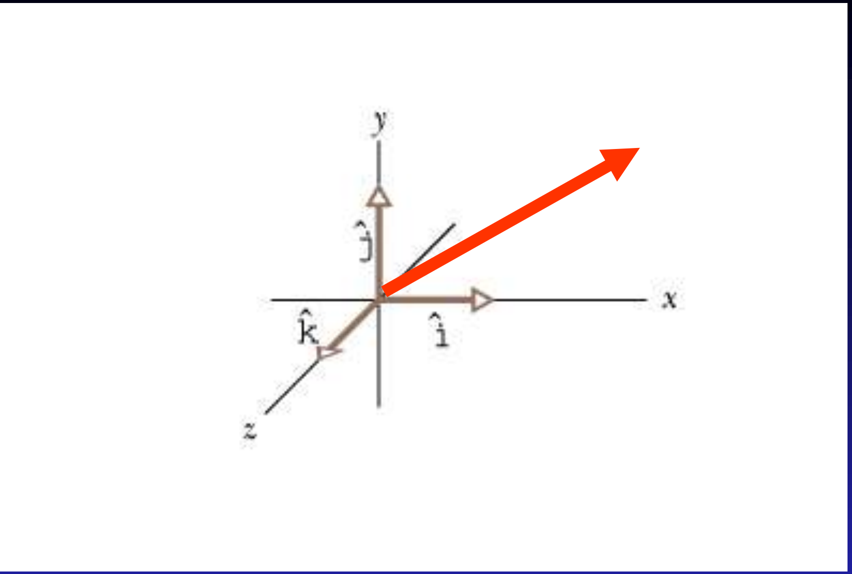
See blackboard

$$\vec{a} = (a_x, a_y) = (4.2, -1.5)$$

$$\vec{a} + \vec{b} = ?$$

$$\vec{b} = (b_x, b_y) = (-1.6, 2.9)$$

Unit Vector Notation



$$\vec{a} = (4.2, -1.5) \text{ or } \vec{a} = 4.2\hat{i} + -1.5\hat{j}$$

$$\vec{b} = (-1.6, 2.9) \text{ or } \vec{b} = -1.6\hat{i} + 2.9\hat{j}$$

Unit vectors have magnitude 1 and are “unitless” … they only give the direction!!!!

VECTOR NOTATION

- Components for a vector may be expressed in unit vector notation
  - $\hat{i}$ is a unit vector in the x direction
  - $\hat{j}$ is a unit vector in the y direction
  - $\hat{k}$ is a unit vector in the z direction
- Bold type or an arrow above the symbol denotes a vector; e.g., **A** or $\vec{A}$
- The magnitude of the above vector is designated A

VECTOR ALGEBRA cont.

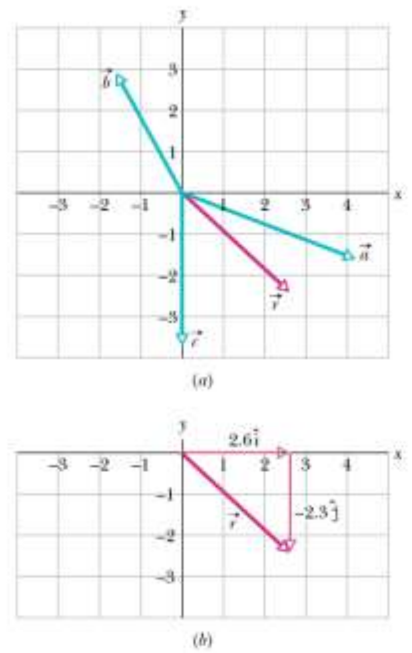
- Vectors are **<u>added or subtracted</u>** according to the rules for ordered pairs

- (a_x,a_y)-(b_x,b_y)-(c_x,c_y)--coordinates!

$$\vec{a} = 4.2\hat{i} + -1.5\hat{j} \text{ or } (4.2, -1.5)$$

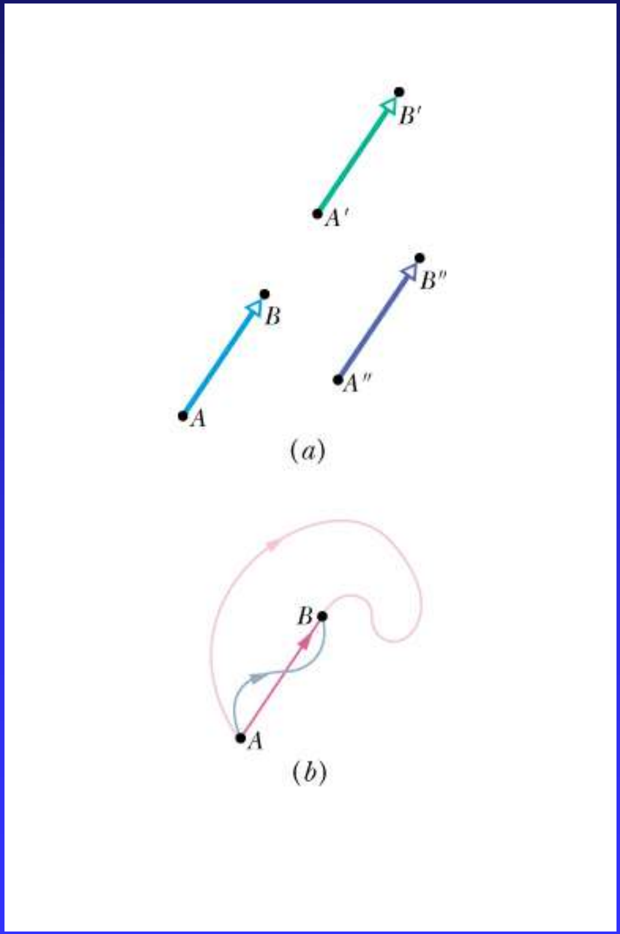
$$\vec{b} = -1.6\hat{i} + 2.9\hat{j} \text{ or } (-1.6, 2.9)$$

$$\vec{c} = 0.0\hat{i} + -3.7\hat{j} \text{ or } (0.0, -3.7)$$



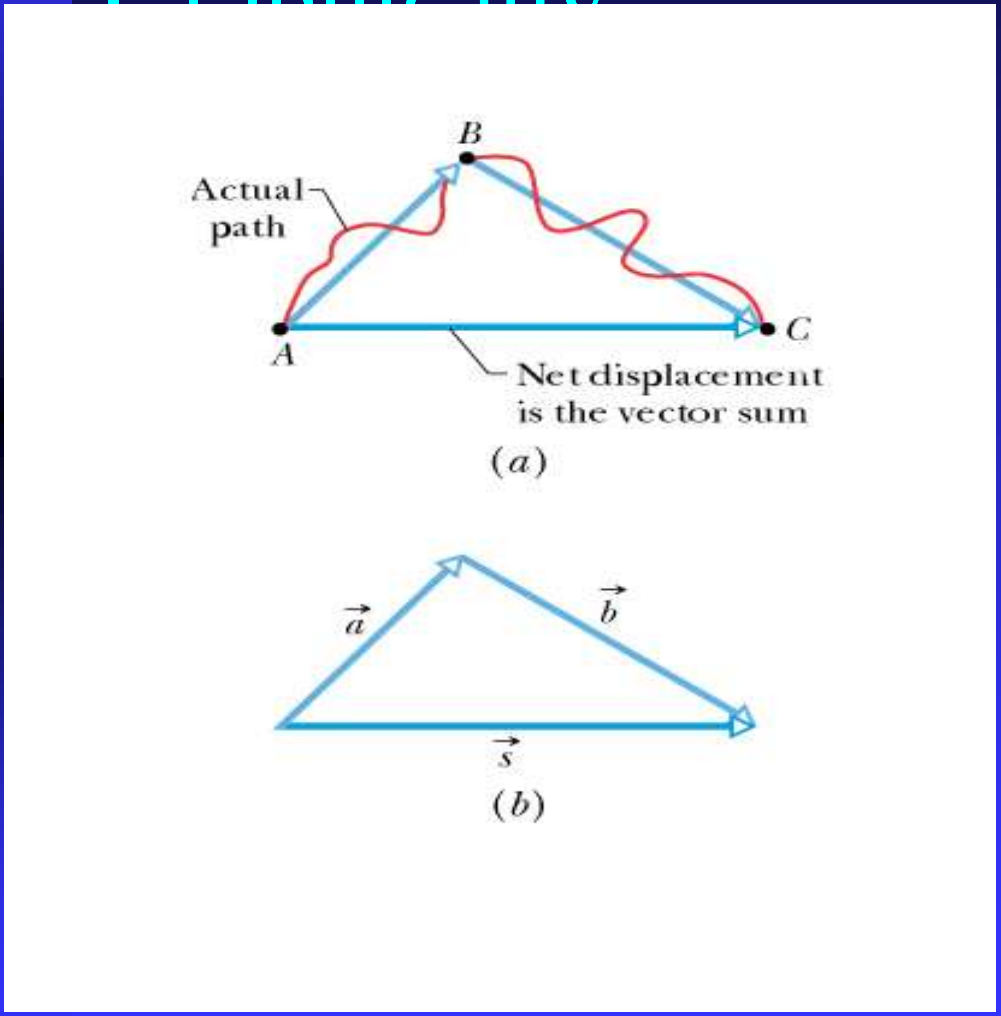
Rule for graphical addition
is implied!!!!

Displacement Vector: now in 2-D



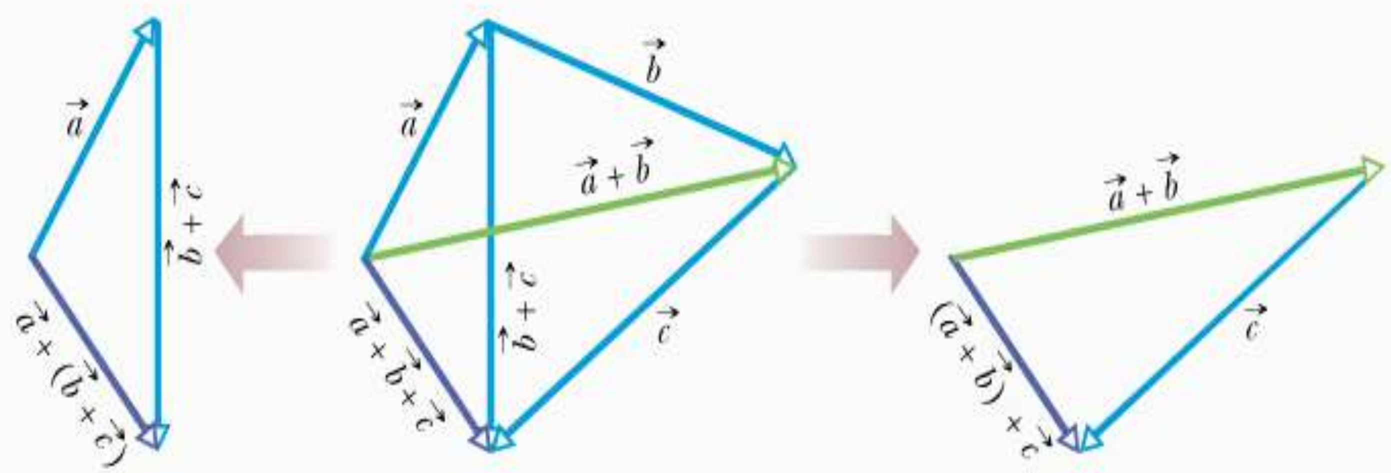
- Displacement
- Three different paths give the same displacement

Addition of displacement vectors Graphically



$$\vec{s} = \vec{a} + \vec{b}$$

Adding more than two vectors graphically



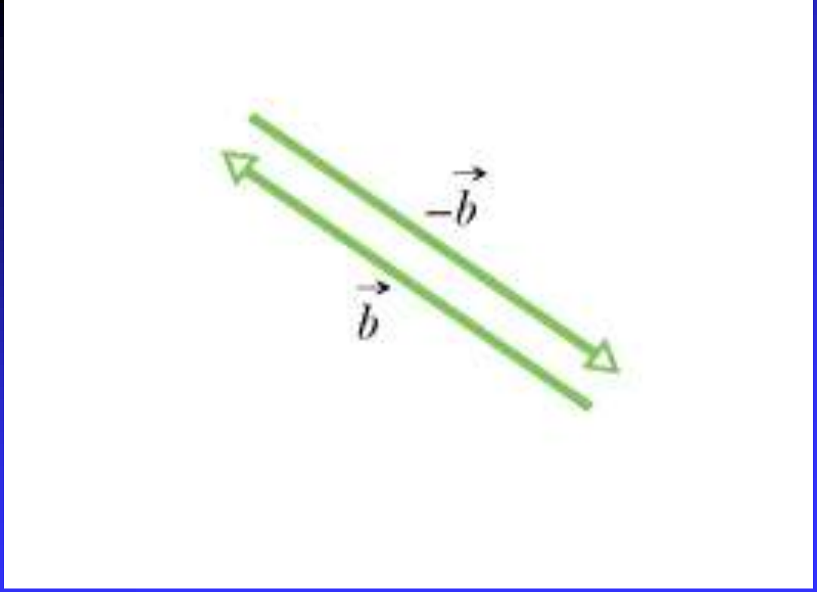
ADDITIONAL VECTOR PROPERTIES

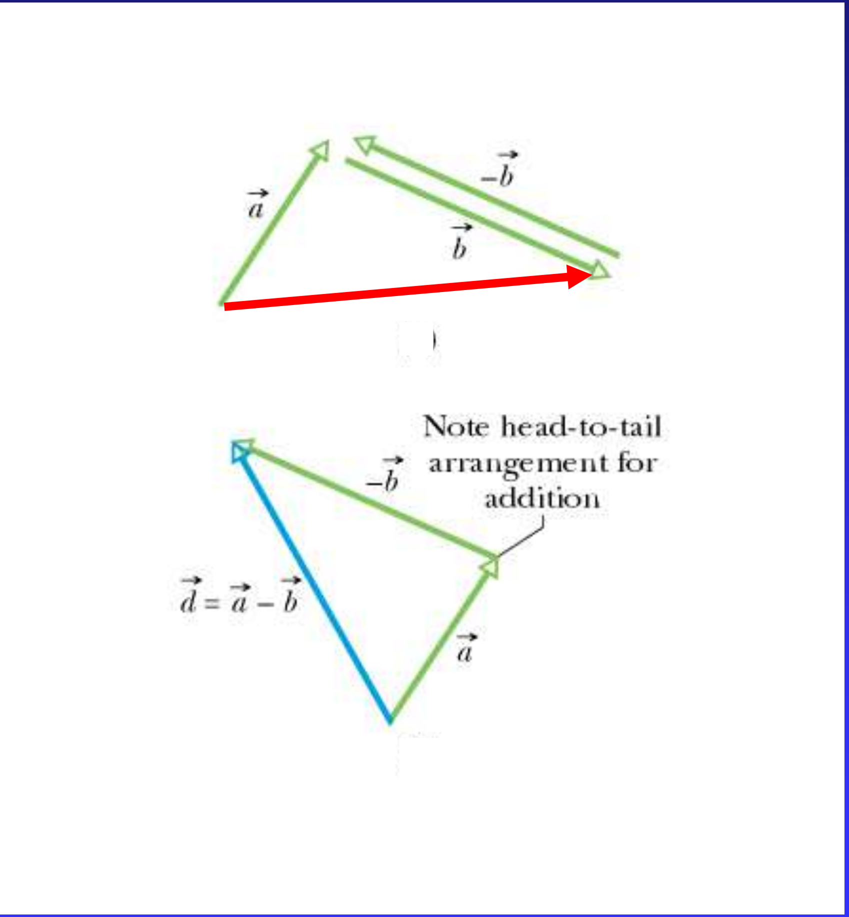
- **A vector can be moved (in a diagram) so long as the magnitude and direction is unchanged**
- Vectors may be expressed as ordered numbers, polar form or in unit vector form
- Vector subtraction may be accomplished by multiplying the subtracted vector by –1 and using the technique for adding

Subtraction of Vectors -graphically

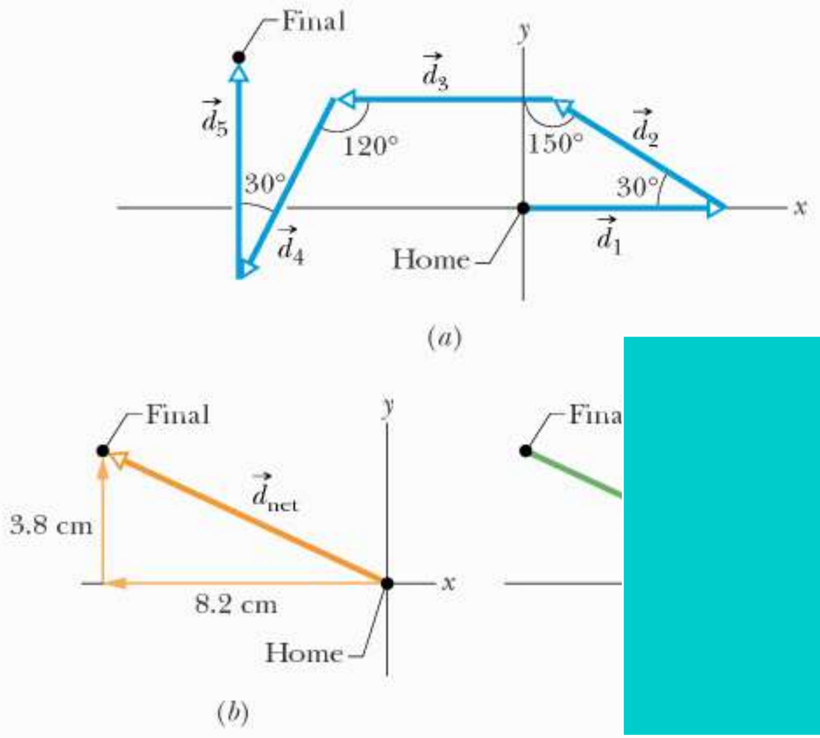
$\vec{d} = \vec{a} + (-\vec{b})$ whereas $\vec{c} = \vec{a} + \vec{b}$

$\vec{d} = \vec{a} - \vec{b}$





Ant Example



Find resultant

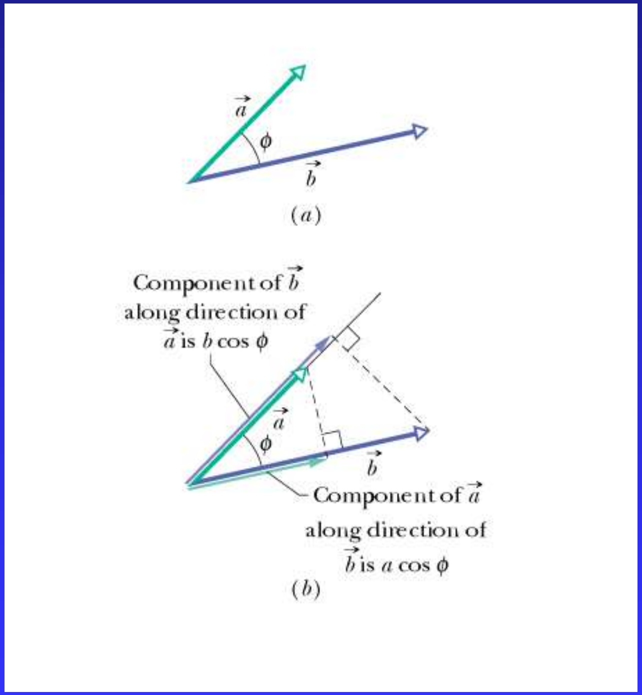
Find the vector home

Vector Multiplication

- Vectors can be multiplied in two ways
  - A dot product of two vectors results in a ***scalar*** $c = \vec{a} \cdot \vec{b}$
  - A cross product of a vector results in another ***vector*** $\vec{c} = \vec{a} \times \vec{b}$
- **Vectors are NEVER divided!**

The Scalar or Dot Product

- Multiplication of two vectors resulting in a scalar



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Example

$$W = \vec{F}\cdot\vec{d},\ \text{for constant force}$$

Some Properties of the Dot Product

- Dot products commute
- The square of a vector
- Unit vector products

$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$

$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{i}\cdot\hat{k}=0$$

EXAMPLE 2

What is the dot product of

$$\vec{A} = 5.0\,\hat{i} + 3.0\,\hat{j}$$

$$\vec{B} = (2.0,\ 155^\circ)$$

The VECTOR PRODUCT or CROSS PRODUCT

- Vector multiplication yielding another vector
- Yields a vector which has a direction determined by the right hand rule
- Yields a vector perpendicular to the plane containing the other two vectors
- The cross product DOES NOT commute

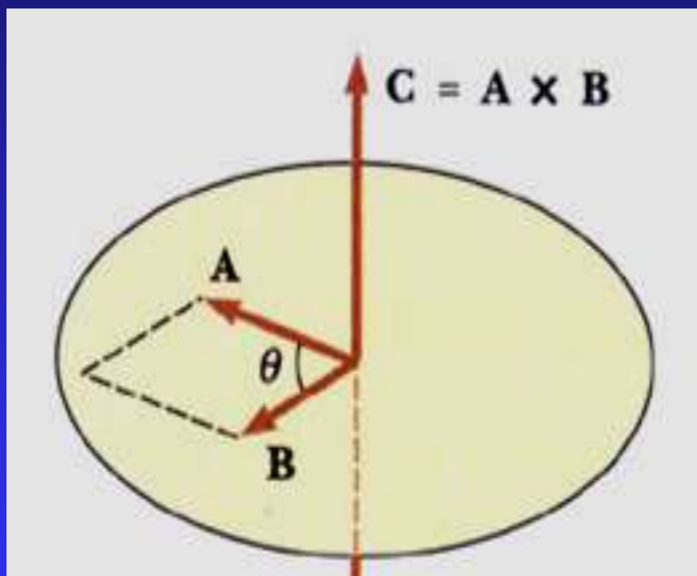
$$\vec{C} = \vec{A} \times \vec{B}$$

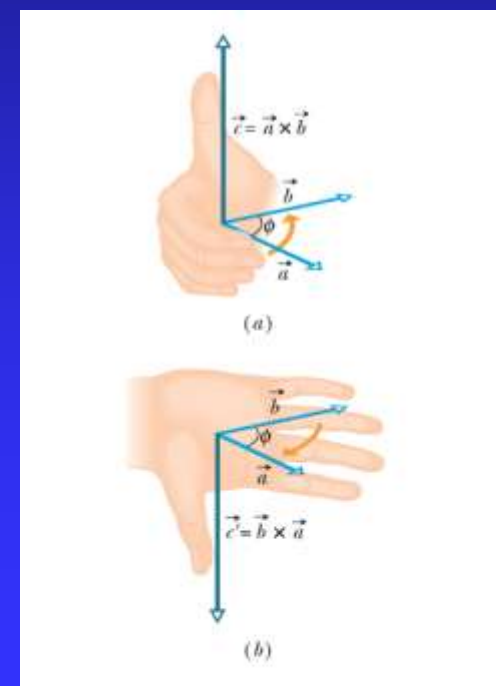
$$-\vec{C} = \vec{B} \times \vec{A}$$

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \quad \textbf{torque}$$

MAGNITUDE OF THE CROSS PRODUCT

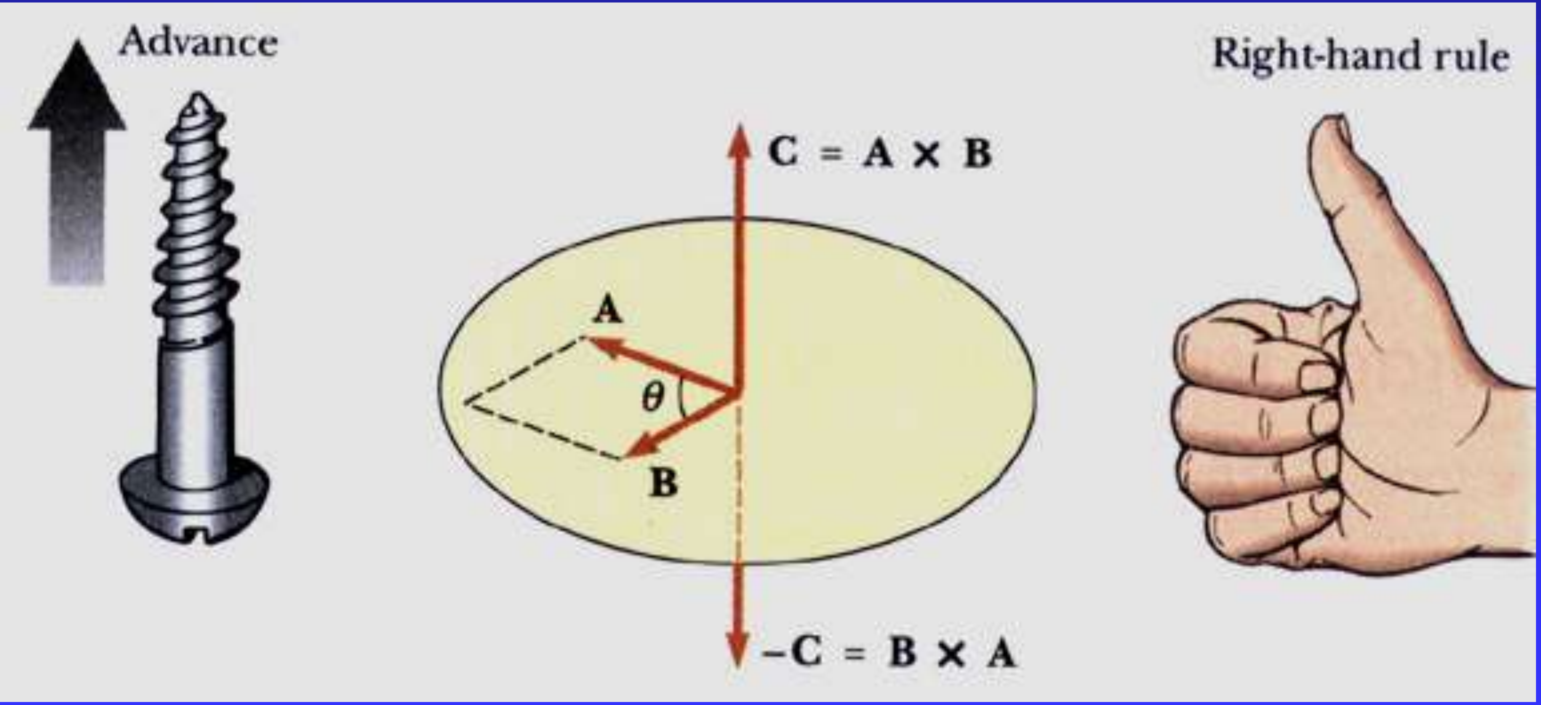
$$\boxed{C = AB\sin\theta}$$





DIRECTION OF THE CROSS PRODUCT

- The right hand rule determines the direction of the cross product



Unit Vector Cross Products

- Using the definition of cross product and righthand rule:

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

$$\hat{i}\times\hat{j}=\hat{k}=-\hat{j}\times\hat{i}$$

$$\hat{j}\times\hat{k}=\hat{i}=-\hat{k}\times\hat{i}$$

$$\hat{k}\times\hat{i}=\hat{j}=-\hat{i}\times\hat{k}$$

EXAMPLE 3

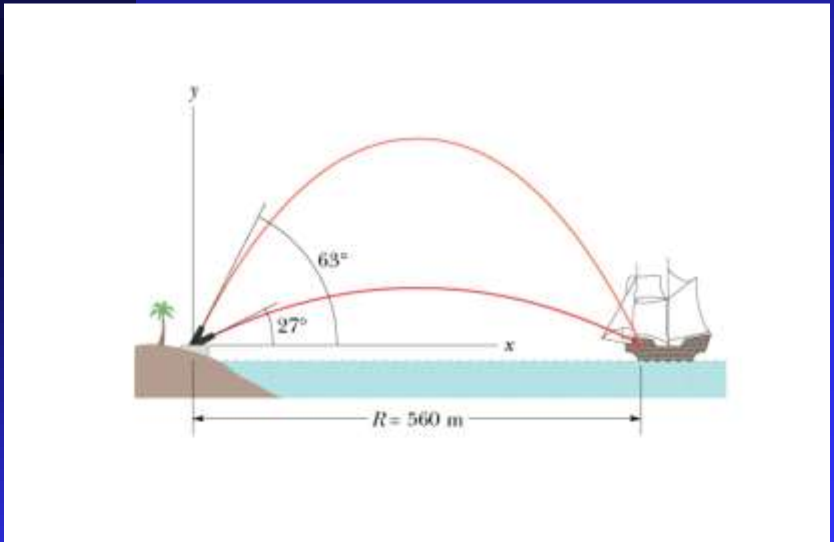
- Calculate the cross product of

$$\vec{A} = 5.00m\hat{i} + 3.00m\hat{j}$$

$$\vec{B} = (2.00m, 155^{o})$$

Projectile Motion

- Classic 2-D problem….Eqs. Of Motion?



$$\Delta x \equiv R = \frac{v_0^2 \sin 2\theta}{g}$$